UNIT-2

HYDROTHERMAL SCHEDULING

OPTIMAL SCHEDULING OF HYDROTHERMAL SYSTEM

- No state or country is endowed with plenty of water sources or abundant coal or nuclear fuel.
- In states, which have adequate hydro as well as thermal power generation capacities, proper co-ordination to obtain a most economical operating state is essential.
- Maximum advantage is to use hydro power so that the coal reserves can be conserved and environmental pollution can be minimized.
- However in many hydro systems, the generation of power is an adjunct to control of flood water or the regular scheduled release of water for irrigation. Recreations centers may have developed along the shores of large reservoir so that only small surface water elevation changes are possible.
- The whole or a part of the base load can be supplied by the run-off river hydro plants, and the peak orthe remaining load is then met by a proper mix of reservoir type hydro plants and thermal plants. Determination of this by a proper mix is the determination of the most economical operating state of a hydro-thermal system. The hydro-thermal coordination is classified into long term co-ordination and short term coordination.

The previous sections have dealt with the problem of optimal scheduling of a power system with thermal plants only. Optimal operating policy in this case can be completely determined at any instant without reference to operation at other times. This, indeed, is the static optimization problem. Operation of a system having both hydro and thermal plants is, however, far more complex as hydro plants have negligible operating cost, but are required to operate under constraints of water available for hydro generation in a given period of time. The problem thus belongs to the realm of dynamic optimization. The problem of minimizing the operating cost of a hydrothermal system can be viewed as one of minimizing the fuel cost of thermal plants under the constraint of water availability (storage and inflow) for hydro generation over a given period of operation.

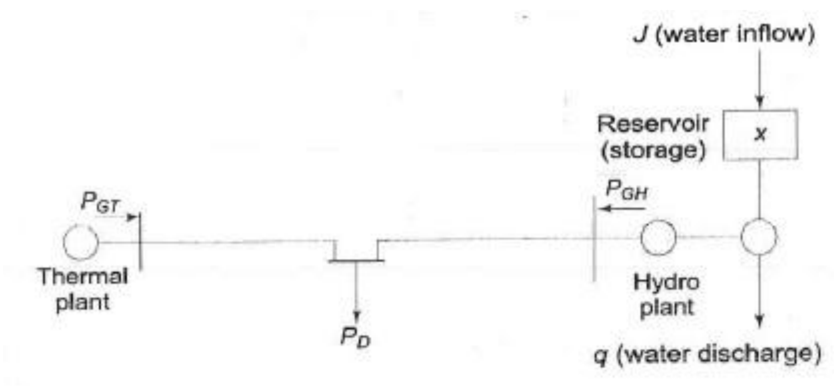

Fig. 2.1 Fundamental hydrothermal system

For the sake of simplicity and understanding, the problem formulation and solution technique are illustrated through a simplified hydrothermal system of Fig. 2.1. This system consists of one hydro and one thermal plant supplying power to a centralized load and is referred to as a fundamental system. Optimization will be carried out with real power generation as control variable, with transmission loss accounted for by the loss formula.

Mathematical Formulation

For a certain period of operation T (one year, one month or one day, depending upon the requirement), it is assumed that (i) storage of hydro reservoir at the beginning and the end of the period are specified, and (ii) water inflow to reservoir (after accounting for irrigation use) and load demand on the system are known as functions of time with complete certainty (deterministic case). The problem is to determine q(t), the water discharge (rate) so as to minimize the cost of thermal generation.

$$C_T = \int_0^T C'\big(P_{GT}(t)\big)dt \qquad (3.1)$$

under the following constraints:

(i) Meeting the load demand

$$P_{GT}(t) + P_{GH}(t) - P_L(t) - P_D(t) = 0;\ t\ \varepsilon\ [0,T] \qquad (3.2)$$

This is called the power balance equation.

(ii) Water availability

$$X'(T) - X'(0) - \int_0^T J(t)dt + \int_0^T q(t)dt = 0 \quad (3.3)$$

where J(t) is the water inflow (rate), X'(t) water storage, and X'(0) , X' (T) are specified water storages at the beginning and at the end of the optimization interval.

(iii) The hydro generation $P_{GH}(t)$ is a function of hydro discharge and water storage (or head), i.e.

$$P_{GH}(t) = f(X'(t),q(t)) \quad (3.4)$$

The problem can be handled conveniently by discretization. The optimization interval T is subdivided into M subintervals each of time length ΔT. Over each subinterval it is assumed that all the variables remain fixed in value. The problem is now posed as

$$min\ \Delta T \sum_{m=1}^{M} C'(P_{GT}^{m}) = min \sum_{m=1}^{M} C(P_{GT}^{m}) \quad (3.5)$$

under the following constraints:

(i) Power balance equation

$$P_{GT}^{m} + P_{GH}^{m} - P_{L}^{m} - P_{D}^{m} = 0 \quad (3.6)$$

where

P_{GT}^{m} = thermal generation in the mth interval

P_{GH}^{m} = hydro generation in the mth interval

P_{L}^{m} =transmission loss in the mth interval

$= B_{TT}(P_{GT}^{m})^2 + 2B_{TH}P_{GH}^{m} + B_{HH}(P_{GH}^{m})^2$

P_{D}^{m} = load demand in the mth interval

(ii) Water continuity equation

$$X'^{m} - X'^{(m-1)} - J^{m}\Delta T + q^{m}\Delta T = 0$$

where

X'^{m} = water storage at the end of the mth interval

J^{m} = water inflow (rate) in the mth interval

q^{m} = water discharge (rate) in the mth interval

The above equation can be written as

$$X^{m} - X^{(m-1)} - J^{m} + q^{m} = 0; m = 1,2, \ldots, M \tag{3.7}$$

where $X^{m} = X'^{m}/\Delta T$ = storage in discharge units.

In Eqs. (3.7), X^{o} and X^{M} are the specified storages at the beginning and end of the optimization interval.

(iii) Hydro generation in any subinterval can be expressed as

$$P_{GH}^{m} = h_{o}\{1 + 0.5e(X^{m} + X^{m-1})\}\,(q^{m} - \rho) \tag{3.8}$$

where

$h_o = 9.81 \times 10^{-3}\, h'_o$

h_o = basic water head (head corresponding to dead storage)

e = water head correction factor to account for head variation with storage

ρ = non-effective discharge (water discharge needed to run hydro generator at no load).

In the above problem formulation, it is convenient to choose water discharges in all subintervals except one as independent variables, while hydro generations, thermal generations and water storages in all subintervals are treated as dependent variables. The fact, that water discharge in one of the subintervals is a dependent variable, is shown below:

Adding Eq. (3.7) for m = l, 2, ...,M leads to the following equation, known as water availability equation

$$X^M - X^0 - \sum_m J^m + \sum_m q^m = 0 \quad (3.9)$$

Because of this equation, only (M - l) qs can be specified independently and the remaining one can then be determined from this equation and is, therefore, a dependent variable. For convenience, q^1 is chosen as a dependent variable, for which we can write

$$q^1 = X^0 - X^M + \sum_m J^m - \sum_{m=2}^{M} q^m \quad (3.10)$$

Solution Technique

The problem is solved here using non-linear programming technique in conjunction with the first order gradient method. The Lagrangian $\mathcal{L}$ is formulated by augmenting the cost function of Eq. (3.5) with equality constraints of Eqs. (3.6)-(3.8) through Lagrange multipliers (dual variables) $\lambda_1^m, \lambda_2^m \ and \ \lambda_3^m$. Thus,

$$\mathcal{L} = \sum_m \Big[C(P_{GT}^m) - \lambda_1^m \big(P_{GT}^m + P_{GH}^m - P_L^m - P_D^m \big) + \lambda_2^m \big(X^m - X^{(m-1)} - J^m + q^m \big) + \lambda_3^m \{ P_{GH}^m - h_o \{ 1 + 0.5 e (X^m + X^{m-1}) \} (q^m - \rho) \} \Big] \quad (3.11)$$

The dual variables are obtained by equating to zero the partial derivatives of the Lagrangian with respect to the dependent variables yielding the following equations

$$\frac{\partial \mathcal{L}}{\partial P_{GT}^m} = \frac{dC(P_{GT}^m)}{dP_{GT}^m} - \lambda_1^m \left(1 - \frac{\partial P_L^m}{\partial P_{GT}^m} \right) = 0 \quad (3.12)$$

$$\frac{\partial \mathcal{L}}{\partial P_G^m} = \lambda_3^m - \lambda_1^m \left(1 - \frac{\partial P_L^m}{\partial P_{GH}^m} \right) = 0 \quad (3.13)$$

$$\left(\frac{\partial \mathcal{L}}{\partial X^m} \right)_{\substack{m \neq M \\ \neq 0}} = \lambda_2^m - \lambda_2^{m+1} - \lambda_3^m \big(0.5 h_o e (q^m - \rho) \big) - \lambda_3^{m+1} \big(0.5 h_o e (q^{m+1} - \rho) \big) = 0 \quad (3.14)$$

and using Eq. (3.7) in Eq. (3.11), we get

$$\frac{\partial \mathcal{L}}{\partial q^1} = \lambda_2^1 - \lambda_3^1 \, h_o \big(1 + 0.5 \, e \, (2X^0 + J^1 - 2q^1 + \rho)\big) = 0 \tag{3.15}$$

The dual variables for any subinterval may be obtained as follows:

(i) Obtain λ_1^m from Eq. (3.12).

(ii) Obtain λ_3^m from Eq. (3.13).

(iii) Obtain λ_2^1 from Eq. (3.15) and other values of λ_2^m $(m \neq 1)$ from Eq. (3.14).

The gradient vector is given by the partial derivatives of the Lagrangian with respect to the independent variables. Thus

$$\left(\frac{\partial \mathcal{L}}{\partial q^m}\right)_{m \neq 1} = \lambda_2^m - \lambda_3^m \, h_o \big(1 + 0.5 \, e \, (2X^{m-1} + J^m - 2q^m + \rho)\big) \tag{3.16}$$

For optimality the gradient vector should be zero if there are no inequality constraints on the control variables.